PHYS1121 - test 1 2003.

PHYS1121 T1, S1, 2003

Q1

a) $x = V_0 \cos\theta \, t$

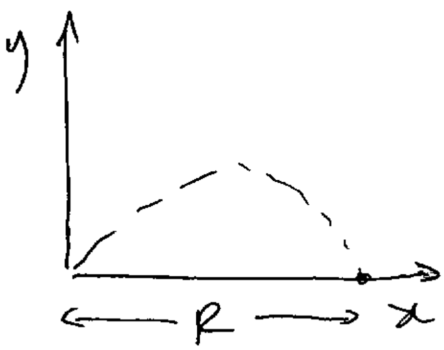
$$t = x / V_0\cos\theta$$

$$y = V_0 \sin\theta \, t - \frac{1}{2} g t^2$$

$$= V_0 \sin\theta \frac{x}{V_0\cos\theta} - \frac{1}{2} g \left(\frac{x}{V_0 \cos\theta}\right)^2$$

$$= (\tan\theta)\, x - \left(\frac{g}{2V_0^2\cos^2\theta}\right) x^2$$

b)



Range $R = x$ when $y = 0$

$$\tan\theta \, R = \left(\frac{g}{2V_0^2\cos^2\theta}\right) R^2$$

$$R = \frac{\sin\theta}{\cos\theta} \frac{2V_0^2\cos^2\theta}{g}$$

$$= \frac{V_0^2}{g} 2\sin\theta\cos\theta$$

$$= \frac{V_0^2 \sin 2\theta}{g}$$

if $\theta = 30^\circ$ $V_0 = 20 \, m.s^{-1}$ $g = 9.8 \, m.s^{-2}$

$$R = \frac{20^2 \sin 60}{9.8} = 35.3 \text{ m}$$

Q1

c) maximum range when

$$R = \frac{V_0^2 \sin 2\theta}{g} \quad \text{is max}$$

ie $\sin 2\theta = 1$, $\theta = 45^\circ$

$$R_{max} = \frac{V_0^2}{g}$$

$$= \frac{20^2}{9.8}$$

$$= 40.8 \text{ m}$$

d) sub into trajectory formula
with $x = \frac{R_{max}}{2} = 20.4$ m.

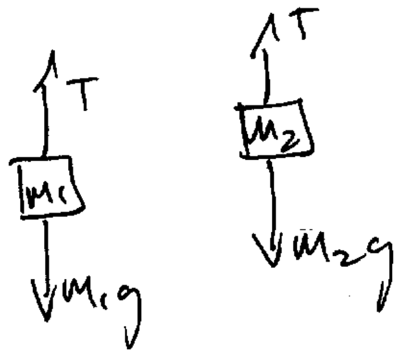
$$y = (\tan 45) 20.4 - \frac{9.8 \quad (20.4)^2}{2 \times 20^2 \times (\cos 45)^2}$$

$$= 10.2 \text{ m}$$

$\therefore$ performer hits the wall

P2

a)



$$\Sigma F_{1y} = m_1 a$$

$$T - m_1 g = -m_1 a$$

$$T = -m_1 a + m_1 g$$

$$\Sigma F_{2y} = m_2 a$$

$$T - m_2 g = m_2 a$$

$$-m_1 a + m_1 g - m_2 g = m_2 a$$

$$a(m_2 + m_1) = g(m_1 - m_2)$$

$$a = \left(\frac{m_1 - m_2}{m_2 + m_1}\right) g$$

$$= \left(\frac{3-5}{5+3}\right) g$$

$$= g/4$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$= 2 \times \frac{g}{4}(0.5)$$

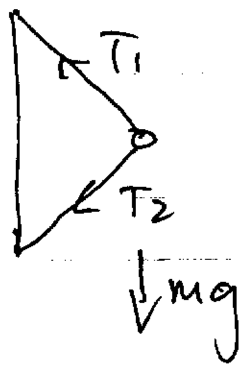
$$v = \sqrt{\frac{g}{4}}$$

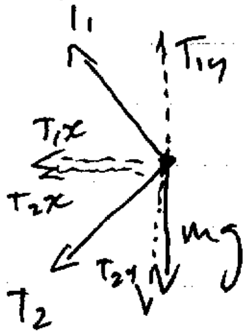
$$= \cancel{2.43}\ m \cdot s^{-1}$$

$$1.56$$

Q2

b) i)





ii) $\Sigma F_y = ma_y$

$$T_{1y} - T_{2y} - mg = 0 \qquad \{a_y = 0\}$$

$$T_1 \cos\theta - T_2 \cos\theta - mg = 0$$

$$T_2 = T_1 - \frac{mg}{\cos\theta}$$

$$T_1 = 70N \quad \theta = 30^\circ$$

$$T_2 = 70 - \frac{2 \times 9.8}{\cos 30}$$

$$= 47.4 \ N$$

iii) $\Sigma F_x = ma_x$

$$T_{1x} + T_{2x} = \frac{mv^2}{r}$$

$$T_1 \sin\theta + T_2 \sin\theta = \frac{mv^2}{r}$$

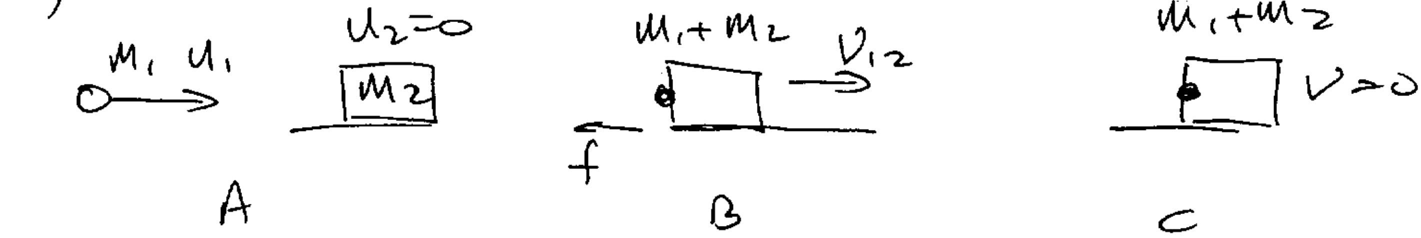
$$v = \left[\frac{r\sin\theta}{m}(T_1 + T_2)\right]^{1/2}$$

$$= \left(\frac{0.75 \times \sin 30}{2} \times (47.4 + 70)\right)^{1/2}$$

$$= 4.7 \ m.s^{-1}$$

Q3

a)



After collision: B to C

$$\Delta K = W$$

$$\tfrac{1}{2}(m_1 + m_2) v_{12}^2 = Fx$$

$$= \mu N x$$

$$= \mu (m_1 + m_2) g x$$

$$v_{12} = \sqrt{2 \mu g x}$$

{alternate solution}

$$F = \mu (m_1 + m_2) g$$

$$= (m_1 + m_2) a$$

$$\therefore a = \mu g$$

$$v_{12}^2 = u_{12} + 2ax$$

$$v_{12} = \sqrt{2 \mu g x}$$

during collision A to B

$$P_i = P_f \quad \text{\{momentum is conserved\}}$$

$$m_1 u_1 = (m_1 + m_2) v_{12}$$

$$u_1 = \left(\frac{m_1 + m_2}{m_1} \right) v_{12}$$

$$= \left(\frac{m_1 + m_2}{m_1} \right) \sqrt{2 \mu g x}$$

$$= \left(\frac{0.06 + 2}{0.06} \right) (2 \times 0.2 \times 9.8 \times 16.4)^{1/2}$$

$$= 275 \text{ m s}^{-1}$$

23

b) KE lost in collision

$\Delta K_{AB} = KE_A - KE_B$

$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2) v_{12}^2$

$= \frac{1}{2} \times 0.06 \times 275^2 - \frac{1}{2}(0.06+2) \times 8^2$

$= 2200$ N

total mechanical energy = $KE_{initial} = KE_A$

$KE_A = \frac{1}{2} m_1 u_1^2 = \frac{1}{2} \times 0.06 \times 275^2$

$= 2268$ N

percentage lost = $\frac{\Delta K_{AB}}{K} = \frac{2200}{2268} = 97\%$

c) $\Delta E_{TOT} = 0 = \Delta K_{AC} - \Delta E_{friction} - \Delta E_{collision}$

percentage lost due to friction = 3%

d) work done by friction force is $\Delta KE_{B \to C}$

$W = KE_B - KE_C$

$= \cancel{\frac{1}{2}(0.06+2)} \; \frac{1}{2}(0.06+2) \times 8^2 - 0$

$= 66$ N

Q4

a) $x = A\cos\omega t$

$$v = \frac{dx}{dt} = -A\omega\sin\omega t$$

$$v^2 = A^2\omega^2\sin^2\omega t$$

$$= \omega^2\left(A^2 - A^2\cos^2\omega t\right)$$

$$= \omega^2\left(A^2 - x^2\right)$$

$$v = \pm\,\omega\sqrt{A^2 - x^2}$$

b) $A = 0.1 \quad v = 1$ when $x = 0.03$.

$$v^2 = \omega^2\left(A^2 - x^2\right)$$

$$\omega^2 = v^2 / \left(A^2 - x^2\right)$$

$$\omega = \sqrt{\frac{1^2}{\left(0.1^2 - 0.03^2\right)}}$$

$$= 10.5 \text{ rad.s}^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10.5}$$

$$= 0.6 \text{ secs}$$

c) $F = -kx$

$$k = \frac{12}{0.03} = 400 \text{ N.m}^{-1}$$

d) $$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$m = \frac{k}{\omega^2} = \frac{400}{10.5^2}$$

$$= 3.63 \text{ kg}.$$

e) $$U = \frac{1}{2}kx^2$$

$$= \frac{1}{2} \times 400 \times 0.1^2 = 2 \text{ J}$$

f) $E_{TOT} = K + U$ is constant.

$K = 0$ at $x = \pm A$

$$E_{TOT} = U_A = \frac{1}{2}kx_A^2$$

$$= \frac{1}{2} \times 400 \times 0.1^2$$

$$= 2 \text{ J}$$

Q5

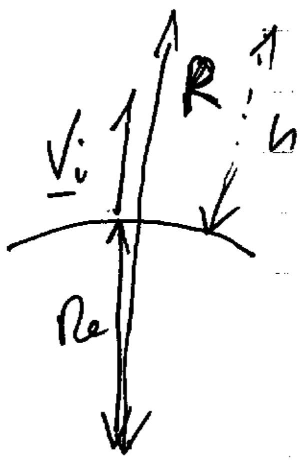
a) $$F = \frac{GM_e m}{(R_e + 1000\times10^3)^2}$$

$$= \left(\frac{6.673\times10^{-11} \times 5.98\times10^{24}}{(6.37\times10^6 + 1000\times10^3)^2}\right) m$$

$$= 7.34\ m$$

$F = ma$ ie $a = 7.34\ \mathrm{m\,s^{-2}}\ (= g)$

b)



$$KE_i + PE_i = KE_f + KE_f$$

$$\frac{1}{2}mv^2 - \frac{GM_e m}{R_e} = 0 + -\frac{GM_e m}{R}$$

$$\frac{1}{R} = \frac{1}{R_e} - \frac{v^2}{2GM_e}$$

$$R = \left(\frac{1}{R_e} - \frac{v^2}{2GM_e}\right)^{-1}$$

$$= \left(\frac{1}{6.37\times10^6} - \frac{8000^2}{2\times6.673\times10^{-11}\times5.98\times10^{24}}\right)^{-1}$$

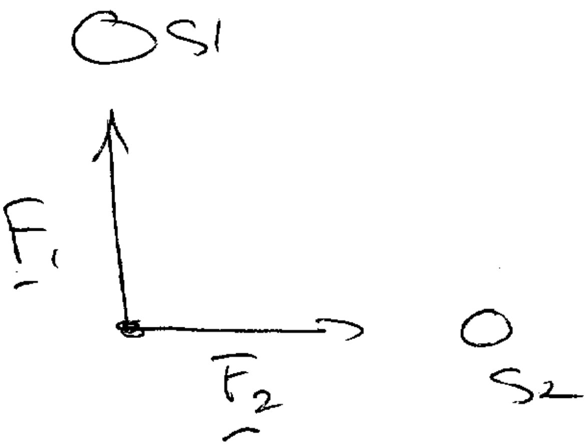
$$= 13\times10^6\ m$$

height $= R - R_e$

$$= 6.6\times10^6\ m.$$

Q5

c)



$$\underline{F}_{TOT} = \underline{F}_{1S} + \underline{F}_{2S}$$

$$= \frac{G M_c M_2}{r_2^2}\hat{i} + \frac{G M_c M_1}{r_1^2}\hat{j}$$

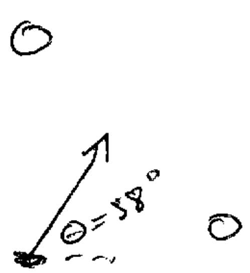
magnitude $$F = \sqrt{\left(\frac{G M_c M_2}{r_2^2}\right)^2 + \left(\frac{G M_c M_1}{r_1^2}\right)^2}$$

$$= \sqrt{\left(\frac{6.67\times10^{-11}\times5\times10^{8}\times8\times10^{30}}{(50\times10^{6})^2}\right)^2 + \left(\frac{6.67\times10^{-11}\times5\times10^{8}\times5\times10^{30}}{(50\times10^{6})^2}\right)^2}$$

$$= \cancel{5\times10^{4}\ N} = 1\times10^{8}\ N$$

in direction $\theta = \tan^{-1}\left(\frac{F_1}{F_2}\right)$

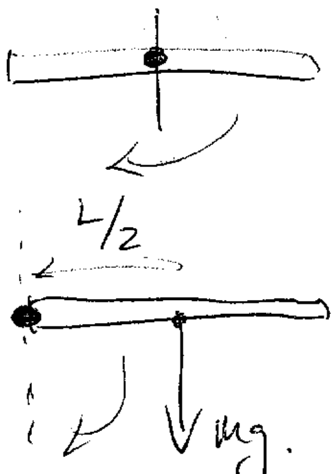
$$= 58^\circ$$



- massive force directs comet towards centre of mass of the two stars.

Q6.

a)



$$I = \frac{1}{12} ML^2$$

$$I = I_{cm} + Mh^2$$

$$= \frac{1}{12} ML^2 + \frac{ML^2}{4}$$

$$= \frac{1}{3} ML^2$$

$$\tau = rF\sin\theta = \frac{l}{2} \cdot mg.$$

$$\Sigma \tau = I\alpha$$

$$\frac{l}{2} mg = \frac{1}{3} ml^2 \alpha$$

$$\alpha = \frac{3}{2}\frac{g}{l} = \frac{3}{2}\frac{9.8}{4} = 3.675 \text{ rad/s}^2$$

b) 1/ linear momentum is constant

$$\Sigma F_{ext} = 0$$

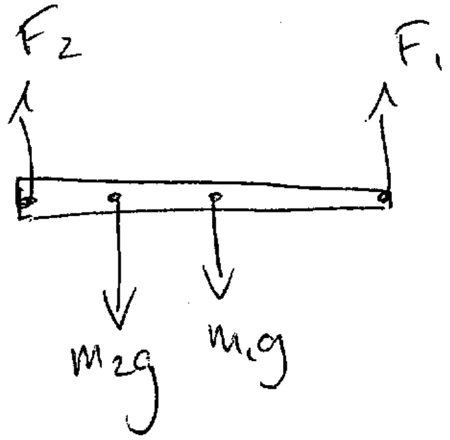
2/ angular momentum is constant

$$\Sigma \tau_{ext} = 0$$

3/ if static $P = L = 0$

Q6.

c)



balance of torque.

$$\Sigma \tau_z = 0.$$

$$-F_1 l + m_1 g \frac{l}{2} + m_2 g \frac{l}{4} = 0$$

$$F_1 = g\left(\frac{m_1}{2} + \frac{m_2}{4}\right)$$

$$= 9.8\left(\frac{5}{2} + \frac{10}{4}\right)$$

$$= 49 \text{ N}$$